In this project, you will devise a numerical method to shed light onto the N-body problem, which is a model for the dynamics of several massive bodies in the presence of a gravitational field. Letting the number of masses be N (hence the name of the problem!), then the aim is to determine the position in three-dimensional space of each body,

$$\boldsymbol{r}_{j} = \begin{bmatrix} (r_{x})_{j} \\ (r_{y})_{j} \\ (r_{z})_{j} \end{bmatrix}, \quad j = 1, \dots, N$$
(1)

The equations of motion that that dictate the dynamics of these masses are

$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = \sum_{\substack{j=1\\j \neq i}}^N \frac{Gm_i m_j (\mathbf{r}_j - \mathbf{r}_i)}{||\mathbf{r}_j - \mathbf{r}_i||^3}$$
(2)

where m_i is the *i*th mass and *G* is the gravitational constant.

1. Select an appropriate numerical method/methods to simulate the *N*-body system dynamics. Then, apply your method(s) to meaningfully chosen values for *N* and the various masses. You *may not* use canned Matlab/Python time stepping routines such as rk45, and must write your solver from scratch.

You can (and are encouraged) to use curiosity to play with the various parameters, but your final choices of parameters should be informed by some research into the problem. For example, there are well-documented examples for N = 2 or N = 3, as well as specific sub-cases of the *N*-body problem (such as the so-called "planetary problem"). For example, what happens if you try to model the dynamics of a satellite in a gravitational field with the earth and the moon? Find questions that keep you up at night and use this project to answer them!

- 2. Write a report that
 - (a) Describes the numerical method(s) you are using. The method should be explained clearly, as should your justification for using this method using arguments of accuracy and stability from the class. The report should also demonstrate that you have correctly implemented the method (e.g., using a convergence test). You may even choose to compare multiple methods against one another.
 - (b) Explains the results from parameters that you choose to explore in detail. There is no one answer for what your exploration should be. You may choose to explain one set of parameters for which the dynamics are especially complex, or ten sets of parameters that are systematically varied to see the effect on some desired output, or something else entirely. Just remember that this is worth 25% of your grade, so select a problem that is a worthy adversary, and provide clearly presented explanations that demonstrate significant understanding of the problem.

Notice that even for N = 3, the dimensions of the system are large enough that it is not obvious how to meaningfully plot results. A crucial piece of being an engineer is conveying technical information in a concise, visually appealing manner. Make sure your report has visually pleasant plots that are easy to read and that clearly convey important information about your system. This information will depend on your project, and could be position of one of your masses as a function of time, mean distance between two bodies as a function of one of the masses that is being changed, *etc.*

(c) Your report should be typed (*e.g.*, using LATEXor Word) in a well-structured format, with equations and figures provided as appropriate. See page 3 for a possible document structure (though this is not mandatory; use the organization that is best for your narrative). The tone should be professional and thoughtful; this is a formal report worth a quarter of your semester grade. There is no page limit, *per se*; just make sure you have convincingly and clearly accomplished points a) and b).

The most important thing to remember is that this is worth 25% of your grade, so I am looking for you to go well beyond what is expected from a homework submission, and take ownership of your work on this project. Going "beyond what is expected" will mean different things to different people. For some of you, that may mean exploring new numerical methods we did not consider in class. For others, it may mean taking a method used in class (and explaining it appropriately), and having the "beyond" part be your exploration into the N-body problem. Either option is great; just be sure to work hard and have fun!

You will work in a small assigned group of students to complete this project. You may divvy up the work as you see fit: every can work equally on every part of the project or you can provide team leads for things like defining the problem statement, picking and justifying the method, writing the report, etc. Whatever of these options you take, be sure that every group member contributes meaningfully to both topics (1) and (2) above. There will be a survey that each student will be asked to fill out after submitting the report, and if there is a consensus that one or more of the team members was not a significant contributor, there will be a substantial grade based penalty depending on the severity of the infraction.

A sample document structure is

1 Introduction

Motivate the *N*-body problem and why it is worth studying, particularly with respect to the specific sub-cases/parameters you will be investigating.

2 The numerical method(s)

2.1 Description/overview

General description/derivation of the method and how you are implementing it.

2.2 Justification for this choice of method(s)

Use accuracy and stability arguments to justify your choice of method.

2.3 Demonstration of correct implementation

Use a convergence test or some other means to convince the reader that you have correctly implemented the method, and that the results you present use a sufficiently small value of Δt .

3 **Results for the** *N***-body problem**

The structure here will be up to your specific investigations, and should have logically designated subsections with clear descriptions of the parameters considered, well-reasoned explanations of the results (with compelling and easy-to-read figures and an appropriate associated discussion).

4 Conclusions

Summarize your choice of method and key findings. Provide an outlook for follow-up investigations, extensions to the method, *etc*.

5 Appendix: code used